1 Adjusted Staking Percentage Formula

Currently "operator share" and "staking percentage" are shown to tokenholders when choosing a staking pool. Considering two metrics simultaneously can be confusing. We are here proposing a single metric called "adjusted staking percentage" that aggregates the combined effects of the "operator share" and "staking percentage" on staking rewards per ZRX. A user, or a simple deterministic algorithm, could therefore just rank staking pools by their adjusted staking percentage (lowest values mean most favorable conditions to receive a higher reward per ZRX).

1.1 Report an 'adjusted staking percentage' that accounts for effects of the 'operator share' and the 'staking percentage' simultaneously

A user could then just pick whichever of the top 5 MMs has the lowest 'adjusted staking percentage.' We could still report the 'operator share' and 'the staking percentage,' but a user would no longer need to look at these stats to decide whom to stake.

In Equation 1, I show a proposed formula that maps the statistics currently displayed on the dashboard: the 'operator share,' θ_i, and the 'staking percentage,' s_i into an 'adjusted staking percentage,' $\widetilde{s}_i$.

$$\widetilde{s}_i = \left(\frac{1-\alpha}{1-\theta_i}\right)^{\frac{1}{\alpha}} s_i \tag{1}$$

To identify the pool with the highest rate of return per staked ZRX, all the user would need to do is pick the pool with the lowest value of $\widetilde{s}_i$. Furthermore, when the system is in equilibrium (i.e. all delegators and MMs are making approximately optimal choices), we should observe that $\widetilde{s}_i \approx s_i \approx 1$. Thus, the adjusted staking percentage can be interpreted in approximately the same way as the original 'staking percentage' statistic.

2 Staking Math To Obtain Equation 1

2.1 Summary

Our first objective is to do the math for ZRX holders, so that they just see an 'adjusted staking percentage' and will know that the pool with the lowest 'adjusted staking percentage' offers the best return.

The ZRX holder would need to look at the MM i's fees generated, t_i, the MMs operator share, θ_i, and the MMs current stake, z_i, to figure out which pool offers the best return. We are going to map (t_i, θ_i, z_i) into an 'adjusted staking percentage' that is inversely related to the pool's expected return.

A second objective is to have the 'adjusted staking percentages' hover around 100% when the system is in a healthy state. To achieve that we need to forecast what the operator shares, θ_i, are likely to be and normalize our 'adjusted staking percentage' using this expected value of θ_i.

The MMs will want to select an operator share, θ_i, that maximize their expected income. If they get greedy and set θ_i too high, then they will not attract enough ZRX and the pool will not generate enough revenue. If they are too generous and set θ_i too low, then the pool will attract lots of ZRX and generate more revenue, but the MMs slice of the fee pie will be too small. There is thus an optimal value of θ_i that we would expect MMs to converge upon.

2.2 Delagator's Choice of Pool

I will assume that the delegator observes the fees that will be generated by pool i, t_i, the stake currently allocated to pool i, z_i, and the operator share of pool i, θ_i. This info is sufficient to allow the delagator to identify the pool with the highest rate of return on staked ZRX.

The total delegator payout of pool i, $r_{i,d}$, is shown in Equation 2, where in addition to the variables previously defined, $\hat{\tau}$ is the total pot of rewards to be distributed, $\hat{t}$ is the total fee earnings of all pools, and $\hat{z}$ is the total ZRX stake of all pools.

$$r_{i,d} = \frac{\hat{\tau}}{\hat{t}^{\alpha}\hat{z}^{1-\alpha}} (1-\theta_i) (t_i)^{\alpha} (z_i)^{1-\alpha} \tag{2}$$

We can rearrange Equation 2 to obtain an expression for pool i's delegator payout per staked ZRX as shown in Equation 3.

$$\frac{r_{i,d}}{z_i} = \frac{\hat{\tau}}{\hat{t}^{\alpha}\hat{z}^{1-\alpha}} (1-\theta_i) \left(\frac{t_i}{z_i}\right)^{\alpha} = \frac{\hat{\tau}}{\hat{z}} (1-\theta_i) \left(\frac{\frac{z_i}{t_i}}{\frac{\hat{z}}{\hat{t}}}\right)^{-\alpha} \tag{3}$$

We can define $s_i = \frac{\frac{z_i}{t_i}}{\frac{\hat{z}}{\hat{t}}}$, and rewrite Equation 3 as shown in Equation 4.

$$\frac{r_{i,d}}{z_i} = \frac{\hat{\tau}}{\hat{z}} \left(\frac{1-\theta_i}{s_i^{\alpha}}\right) \tag{4}$$

Note, that s_i is our current definition of a pool's 'staking percentage' of pool i. Furthermore, note that, holding θ_i fixed, $\frac{r_{i,d}}{z_i}$ is strictly decreasing in s_i for all $\alpha > 0$. This means that, if all pool's adopt the same operator share, then the delegator would want to pick whichever pool has the lowest staking percentage.

But what if both θ_i and s_i vary across pools? To simplify this case, we could rewrite Equation 4 in terms of an adjusted staking percentage, $\widetilde{s}_i = \left(\frac{1-c}{1-\theta_i}\right)^{\frac{1}{\alpha}} s_i$ and a normalizing constant, $c \in (-\infty, 1)$, as shown in Equation 5.

$$\frac{r_{i,d}}{z_i} = \frac{\hat{\tau}}{\hat{z}} \left(\frac{1}{1-c}\right)^{\frac{1}{\alpha}} \left(\frac{1}{\widetilde{s}_i^{\alpha}}\right) \tag{5}$$

Note that now, even if both θ_i and s_i vary simultaneously, the delegator would always want to pick the pool with the lowest adjusted staking percentage,

$\widetilde{s}_i$. Note also that we can pick any value of $c \in (-\infty, 1)$ we want and still preserve this property. However, ideally we would want to set c close to the operator's optimal choice of θ_i, so that when the system is functioning as expected (i.e. in equilibrium), $\frac{1-c}{1-\theta_i} \approx 1$ and $\widetilde{s}_i \approx s_i \approx 1$.

2.3 Operator's Choice of θ_i

In equilibrium, delegators are expected to equalize the returns to ZRX stake across pools, so that for all i, $\frac{r_{i,d}}{z_i} = \frac{\hat{\tau}}{\hat{t}^\alpha \hat{z}^{1-\alpha}} (1-\theta_i) \left(\frac{t_i}{z_i}\right)^\alpha = \hat{k}$ where $\hat{k}$ is a constant rate of reward per ZRX staked. If we arrange this expression as shown in Equation 6, we can obtain a formula that maps the pool operator attributes θ_i and t_i into an implied amount of staked ZRX, z_i, that the pool will receive.

$$z_i(\theta_i, t_i) = \left(\frac{\hat{\tau}}{\hat{t}^\alpha \hat{z}^{1-\alpha} \hat{k}}\right)^{\frac{1}{\alpha}} (1-\theta_i)^{\frac{1}{\alpha}} t_i \tag{6}$$

An expression for the operator's reward, $r_{i,mm}$, is shown in Equation 7.

$$r_{i,d} = \frac{\hat{\tau}}{\hat{t}^\alpha \hat{z}^{1-\alpha}} \theta_i t_i^\alpha z_i^{1-\alpha} \tag{7}$$

If we substitute Equation 6 into Equation 7, we obtain the expression for the operator's reward as a function of his attributes θ_i and t_i as shown in Equation 8.

$$r_{i,d}(\theta_i, t_i) = \left(\frac{\hat{\tau}}{\hat{t}^\alpha \hat{z}^{1-\alpha} \hat{k}}\right)^{\frac{1}{\alpha}} (1-\theta_i)^{\frac{1-\alpha}{\alpha}} \theta_i t_i \tag{8}$$

The operator is then going to choose a value of θ_i to maximize his expected rewards. In this calculation, we will assume that the operator is too small to affect attributes of the entire network, so that $\left(\frac{\hat{\tau}}{\hat{t}^\alpha \hat{z}^{1-\alpha} \hat{k}}\right)^{\frac{1}{\alpha}}$ is constant. Furthermore, we will assume that the reward payments are too small to meaningfully alter the operator's trading fee generation, so that t_i can be treated as constant as well.[1] Under these assumptions, maximizing Equation 8 is equivalent to maximizing the concave function $(1-\theta_i)^{\frac{1-\alpha}{\alpha}} \theta_i$. To do this we just take the first derivative and set it equal to 0 to obtain the expression for the optimal operator share shown in Equation 9.

$$\theta^* = \alpha \tag{9}$$

By setting the normalizing constant c equal to $\theta^* = \alpha$, we obtain the suggested formula for the adjusted staking percentage shown in Equation 1.

[1] Neither of these assumptions are 100% accurate. However, these types of effects are going to be negligible, so that ignoring them won't affect the answer.